1. Define the concept of a group.

Solution. A group is a set \( G \) together with a binary operation \( \cdot \) satisfying the following conditions:
(i) There exists \( e \in G \) such that \( g \cdot e = g = e \cdot g \), for all \( g \in G \).
(ii) For all \( g \in G \), there exists \( g^{-1} \in G \) such that \( g \cdot g^{-1} = e = g^{-1} \cdot g \).
(iii) For all \( g_1, g_2, g_3 \in G \), \((g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)\).

2. Let \( \tau = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \) and \( \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \) belong to the symmetric group \( S_3 \). Recall that \( \tau^3 = I = \sigma^2 \) and \( \sigma \tau = \tau^2 \sigma \), where \( I \) denotes the identity element of \( S_3 = \{I, \tau, \tau^2, \sigma, \tau \sigma, \tau^2 \sigma\} \). Write the following product as an element of \( S_3 \) in standard form.

\[
\sigma^{11} \tau^5 \sigma^4 \tau^2 \sigma \tau =
\]

Solution.

\[
\begin{align*}
\sigma^{11} \tau^5 \sigma^4 \tau^2 \sigma \tau &= \sigma \tau^2 e e \sigma \tau \\
&= \tau \sigma \tau^2 \sigma \\
&= \tau \tau \sigma \sigma \\
&= \tau^2.
\end{align*}
\]