SPRING 2022: MATH 290 HOMEWORK

The page and section numbers listed below correspond to those in our textbook.

Homework 1. Online homework and Nicholson, Section 1.1: 1b, 2b, 3, 4, 5, 6.

Homework 2. Online homework and Nicholson, Section 1.1: 9-13, Section 1.2, 2b, 4b.

Homework 3. Online homework and Nicholson, Section 1.2: 1, 4a, 5b, 5f, 7b.

Homework 4. Online homework and Nicholson, Section 1.3: 2a, 2b, 5a, 5b, 5c, 5d.

Homework 5. Online homework and Nicholson, Section 2.1: 2, 3, 5.

Homework 6. Online homework and Nicholson, Section 2.3: 2, 4, 5,

Homework 7. Online homework and Nicholson, Section 2.4: 1c, 2f, 3b, 7, 8b

Homework 8. Online homework and Nicholson Section 2.5: 1b, 1d, 1f, 2b, 2d, 2f, 6b, 8b.

Homework 9. Online homework and Nicholson, Section 3.1: 1b, 1 1p, 1n, 5b, 5c, 5d

Homework 10. Online homework and Nicholson, Section 3.2: 1b, 7b, 8b, 8d; Section 3.3: 1a, 1b, 1c, 1d.

Homework 11. Online homework and Nicholson, Section 3.3: 1b,d,f,h.

Homework 12. Online homework and Nicholson, Section 3.3: 5, 8a, 8b, 9a and calculate e^A, for 8a, 8b. Section 3.4: 1b, 1d.

Homework 13. Online homework and Nicholson, Section 3.4: 1c. Section 3.5: 1a, 1b. Solve the problems 1a, 1b as we did in class by finding the matrix e^{At}, where A is the coefficient matrix for the given system of linear differential equations. Note that the solution, in matrix form, is given by $\mathbf{X}(t) = e^{At} \cdot \mathbf{X}(0)$, where $\mathbf{X}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$, $\mathbf{X}(0) = \begin{bmatrix} f_1(0) \\ f_2(0) \end{bmatrix}$ and $e^{At} = Pe^{Dt}P^{-1}$, where D is the diagonal matrix whose entries are the eigenvalues of A and P is the matrix such that $A = PDP^{-1}$.

Homework 14. Nicholson: Use the exponential matrix to solve Section 3.5: 1c, 1d.

Homework 15. Online homework and Nicholson, Section 5.1: 1a,1c,1,e, 2a, 2b.

Homework 16. Online homework and Nicholson, Section 5.2: 1a,b,c,d.

Homework 17. Online homework and Nicholson, Section 5.2: 2a,b,c,d.

Homework 18. Online homework and Nicholson, Section 5.2: 6a,b,c,e.

Homework 19. Online homework and Nicholson, Section 5.2: 3a,b,c,d

Homework 20. 1. For the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, find an orthogonal basis for span$\{v_1, v_2\}$.

2. Find an orthonormal basis for the space spanned by the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$. Then use the dot product in $\mathbb{R}^4$ to find $\alpha, \beta \in \mathbb{R}$ such that $w = \alpha v_1 + \beta v_2$, for $w = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Homework 20. Online homework and Nicholson, Section 8.1: Find orthogonal bases for the spaces given in 1a, 1c, 2a, 2b.

Homework 21. Online homework and Nicholson, Section 8.1: Find orthonormal bases for the spaces given in 1b, 1d, 2d.

Homework 22. 1. For the vectors $\vec{w_1} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ and $\vec{w_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, show that the vector $\vec{v} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$ belongs to the span of $\vec{w_1}$, $\vec{w_2}$ and use the dot product formula to write $\vec{v}$ as a linear combination of $\vec{w_1}, \vec{w_2}$. It is important to note that $\vec{w_1}$ and $\vec{w_2}$ are orthogonal.

2. For $\vec{w}_1, \vec{w}_2$ as in problem 1, let U denote the subspace of $\mathbb{R}^3$ spanned by $\vec{w}_1, \vec{w}_2$ and A denote the matrix whose columns are $\vec{w}_1, \vec{w}_2$. Set $\vec{b} = \begin{bmatrix} 2 \\ -6 \\ -2 \end{bmatrix}$,

(i) Show that $\vec{b}$ is not in U. Equivalently, the system of equations $A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \vec{b}$ has no solution.

(ii) Calculate $\mathbf{p}_U(\vec{b})$, the orthogonal projection of $\vec{b}$ onto U.

(iii) Find the best approximation to a solution to the system of equations $A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \vec{b}$, first, by using $\mathbf{p}_U(\vec{b})$, then by using A^t.