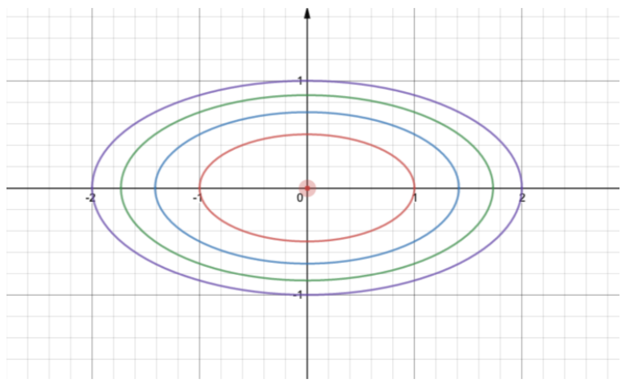
SPRING 2021: MATH 147 QUIZ 1 SOLUTIONS

Each question is worth 5 points. You must justify your answer to receive full credit.

1. Describe in words and sketch the level curves for $f(x,y) = x^2 + 4y^2$, for $c = 1, 2, 3, 4$.

   Solution. The graphs are ellipses with increasing axes.



2. For $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6+2y^2}$, show that the limit along any line through the origin exists and equals zero, but if we take the limit along the curve $y = x^3$, the limit is not zero. What conclusion can you draw from this?

Solution. Taking the limit along the line $y = mx$, we obtain

$$\lim_{x\to 0} \frac{x^3(mx)}{x^6 + 2(mx)^2} = \lim_{x\to 0} \frac{mx^4}{x^6 + 2m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^4 + 2m^2} = \frac{0}{2m^2} = 0.$$

Taking the limit along $y = x^3$, we obtain

$$\lim_{x\to 0} \frac{x^3 \cdot x^3}{x^6 + 2(x^3)^2} = \lim_{x\to 0} \frac{x^6}{3x^6} = \frac{1}{3}.$$

Thus, $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6+2y^2}$ does not exist.

3. Determine if the function $f(x,y) = \begin{cases} \frac{x^3+x^2+xy^2+y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}$ is continuous at (0,0).

Solution. $\lim_{(x,y)\to(0,0)} f(x,y)) = \lim_{(x,y)\to(0,0)} \frac{x(x^2+y^2)+x^2+y^2}{x^2+y^2} = \lim_{(x,y)\to(0,0)}(x+1) = 1$. On the other hand, $f(0,0) = 2$. Therefore, $f(x,y)$ is not continuous at $(0,0)$.